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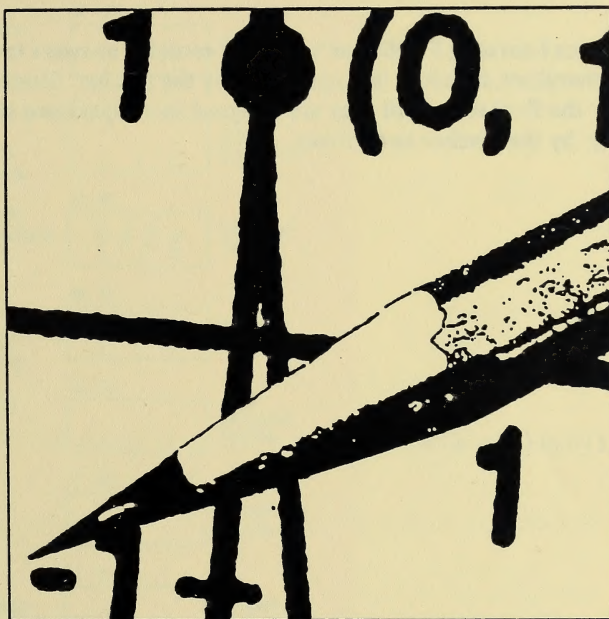
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MATHEMATICS 3

LEARNING FACILITATOR'S MANUAL



UNIT 6: ALGEBRAIC VECTORS AND THEIR APPLICATION



Distance
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Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final tests until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

Mathematics 31
Learning Facilitator's Manual
Unit 6
Algebraic Vectors and Their Application
Alberta Distance Learning Centre
ISBN No. 0-7741-0169-5

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Topic 1: Length of a Vector

- ② 1. Find the distance between the points $A(3, -2)$ and $B(-1, -3)$.

$$\begin{aligned} d &= \sqrt{(3+1)^2 + (-2+3)^2} \\ &= \sqrt{16+1} \\ &= \sqrt{17} \end{aligned}$$

- ③ 2. Find the length of the vector represented by the line segment from $M(3, -2, 1)$ to $N(5, 0, 6)$.

$$\begin{aligned} d &= \sqrt{(5-3)^2 + (0+2)^2 + (6-1)^2} \\ &= \sqrt{4+4+25} \\ &= \sqrt{33} \end{aligned}$$

5

3. Determine if the three points $P(0, 1, 0)$, $Q(2, 3, 1)$, and $R(4, 5, 2)$ are collinear.

$$PQ = \sqrt{(2-0)^2 + (3-1)^2 + (1-0)^2}$$

$$= \sqrt{9}$$

$$= 3$$

$$QR = \sqrt{(4-2)^2 + (5-3)^2 + (2-1)^2}$$

$$= \sqrt{9}$$

$$= 3$$

$$PR = \sqrt{(4-0)^2 + (5-1)^2 + (2-0)^2}$$

$$= \sqrt{36}$$

$$= 6$$

$$PR = PQ + QR$$

Therefore, P , Q , and R are collinear.

5

4. Determine if the three points $A(3, 5)$, $B(0, 4)$, and $C(1, 1)$ form a right triangle.

$$AB = \sqrt{(3-0)^2 + (5-4)^2}$$

$$= \sqrt{10}$$

$$BC = \sqrt{(1-0)^2 + (1-4)^2}$$

$$= \sqrt{10}$$

$$AC = \sqrt{(3-1)^2 + (5-1)^2}$$

$$= \sqrt{20}$$

Since $AC^2 = AB^2 + BC^2$, $\triangle ABC$ is a right triangle.

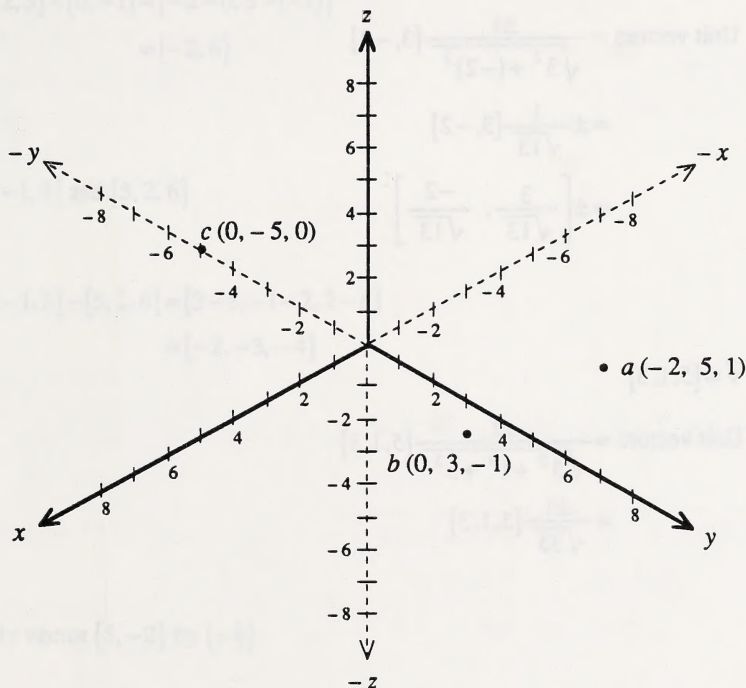
③

5. Plot the following points.

a. $(-2, 5, 1)$

b. $(0, 3, -1)$

c. $(0, -5, 0)$



②

6. a. The characteristic of a point on the xz -plane is $y = 0$.b. The characteristic of a point on the z -axis is $x = y = 0$.

Topic 1

_____ marks

Topic 2: Operations Defined on Algebraic Vectors

- ④ 1. Find two unit vectors collinear with each of the following vectors.

a. $\vec{v} = [3, -2]$

$$\begin{aligned}\text{Unit vectors} &= \frac{\pm 1}{\sqrt{3^2 + (-2)^2}} [3, -2] \\ &= \pm \frac{1}{\sqrt{13}} [3, -2] \\ &= \pm \left[\frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right]\end{aligned}$$

b. $\vec{v} = [5, 1, 3]$

$$\begin{aligned}\text{Unit vectors} &= \frac{\pm 1}{\sqrt{5^2 + 1^2 + 3^2}} [5, 1, 3] \\ &= \frac{\pm 1}{\sqrt{35}} [5, 1, 3]\end{aligned}$$

- ④ 2. Find the sums of the following vectors.

a. $[-2, 5]$ and $[3, -7]$

$$\begin{aligned}[-2, 5] + [3, -7] &= [-2 + 3, 5 - 7] \\ &= [1, -2]\end{aligned}$$

b. $[0, 5, 1]$ and $[3, 2, -8]$

$$\begin{aligned}[0, 5, 1] + [3, 2, -8] &= [0 + 3, 5 + 2, 1 - 8] \\ &= [3, 7, -7]\end{aligned}$$

- ④ 3. Subtract the second vector from the first vector.

a. $[-2, 5]$ and $[0, -1]$

$$\begin{aligned} [-2, 5] - [0, -1] &= [-2 - 0, 5 - (-1)] \\ &= [-2, 6] \end{aligned}$$

b. $[3, -1, 2]$ and $[5, 2, 6]$

$$\begin{aligned} [3, -1, 2] - [5, 2, 6] &= [3 - 5, -1 - 2, 2 - 6] \\ &= [-2, -3, -4] \end{aligned}$$

- ② 4. Multiply vector $[5, -2]$ by $(-\frac{2}{3})$.

$$\left(-\frac{2}{3}\right)[5, -2] = \left[-\frac{10}{3}, \frac{4}{3}\right]$$

5

5. Find x and y if $5[x, 3] + 2[-3, y] = [4, 1]$.

$$5x - 6 = 4$$

$$5x = 10$$

$$x = 2$$

$$15 + 2y = 1$$

$$2y = -14$$

$$y = -7$$

5

6. Prove that $\{[0, 5] + [-3, 1]\} + [-5, -2] = [0, 5] + \{[-3, 1] + [-5, -2]\}$.

LS	RS
$[-3, 6] + [-5, -2]$	$[0, 5] + [-8, -1]$
$[-8, 4]$	$[-8, 4]$
LS	= RS

- ⑥ 7. Determine x , y , and z if $2[3, y, 0] - 5[x, -2, 2] + [1, 3, -4] = [y, z, x]$.

$$6 - 5x + 1 = y$$

$$5x + y - 7 = 0 \quad \textcircled{1}$$

$$2y + 10 + 3 = z$$

$$2y - z + 13 = 0 \quad \textcircled{2}$$

$$0 - 10 + (-4) = x$$

$$x = -14 \quad \textcircled{3}$$

Substitute $\textcircled{3}$ in $\textcircled{1}$.

$$5(-14) + y - 7 = 0$$

$$y = 77$$

Substitute $y = 77$ in $\textcircled{2}$.

$$2(77) - z + 13 = 0$$

$$z = 154 + 13$$

$$= 167$$

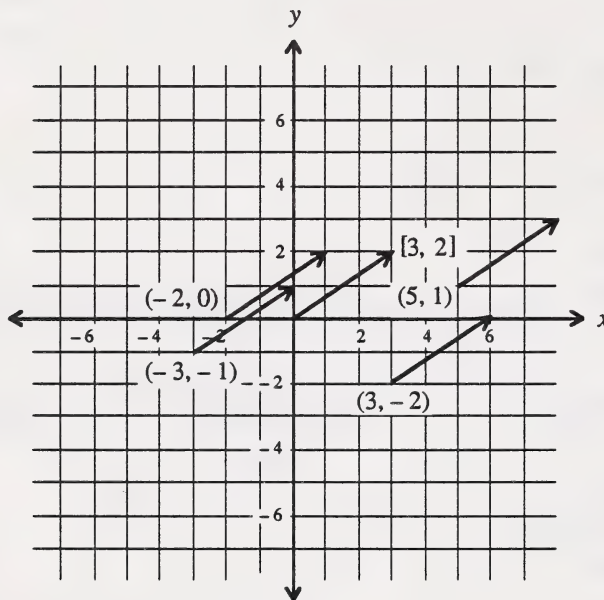
Topic 2

_____ marks

Topic 3: Two- and Three-Dimensional Vectors in Algebraic Form

④

1. If the position (basic) vector is $[3, 2]$, draw an equivalence class of vectors with the following initial points.
- a. $(5, 1)$
- b. $(-2, 0)$
- c. $(-3, -1)$
- d. $(3, -2)$



②

2. Find the position (basic) vector which corresponds to the geometric vector \overrightarrow{AB} determined by $A(3, 7)$ and $B(5, -2)$.

The coordinates of this position (basic) vector are $[5-3, -2-7] = [2, -9]$.

②

3. Find the coordinates of the position (basic) vector which corresponds to the geometric vector \overrightarrow{xy} determined by x which is at $(3, 2, -5)$ and y which is at $(-1, 0, -2)$.

The coordinates of this position (basic) vector are $[-1 - 3, 0 - 2, -2 + 5] = [-4, -2, 3]$.

②

4. $[3, -1]$ is a position (basic) vector. Find the terminal point of vector \overrightarrow{PQ} which is equivalent to the position (basic) vector. The initial point P is $(4, 5)$.

The terminal point is $(4 + 3, -1 + 5) = (7, 4)$.

Topic 3

_____ marks

Topic 4: Collinear and Coplanar Algebraic Vectors

②

1. Find the set of vectors which is parallel to each of the following:

a. $[-5, 1]$

b. $[3, 7, -2]$

$$\{k[-5, 1] \mid k \in R\}$$

$$\{k[3, 7, -2] \mid k \in R\}$$

④

2. Find the set of vectors which is parallel to vector \overrightarrow{PQ} if P and Q are the following points:

a. $P(3, -2) \quad Q(6, 4)$

$$\overrightarrow{PQ} = [6 - 3, 4 + 2] = [3, 6]$$

Thus, the set of vectors is $\{k[3, 6] \mid k \in R\}$.

b. $P(3, 5, -1) \quad Q(0, 2, 4)$

$$\overrightarrow{PQ} = [0 - 3, 2 - 5, 4 + 1] = [-3, -3, 5]$$

Thus, the set of vectors is $\{k[-3, -3, 5] \mid k \in R\}$.

④

3. $[4, 3, -6]$ and $[2, \frac{3}{2}, -3]$ are two vectors. Are they collinear? Explain.

Yes, they are collinear because $[4, 3, -6] = 2[2, \frac{3}{2}, -3]$. They are scalar multiples of each other.

⑧

4. $[3, 5, k]$ and $[-6, l, 4]$ are two collinear vectors. Find k and l .

$$\begin{aligned} -6 &= (-2)(3) \\ [-6, l, 4] &= (-2)[3, 5, k] \\ &= [-6, -10, -2k] \end{aligned}$$

$$\therefore l = -10$$

$$-2k = 4$$

$$\therefore k = -2$$

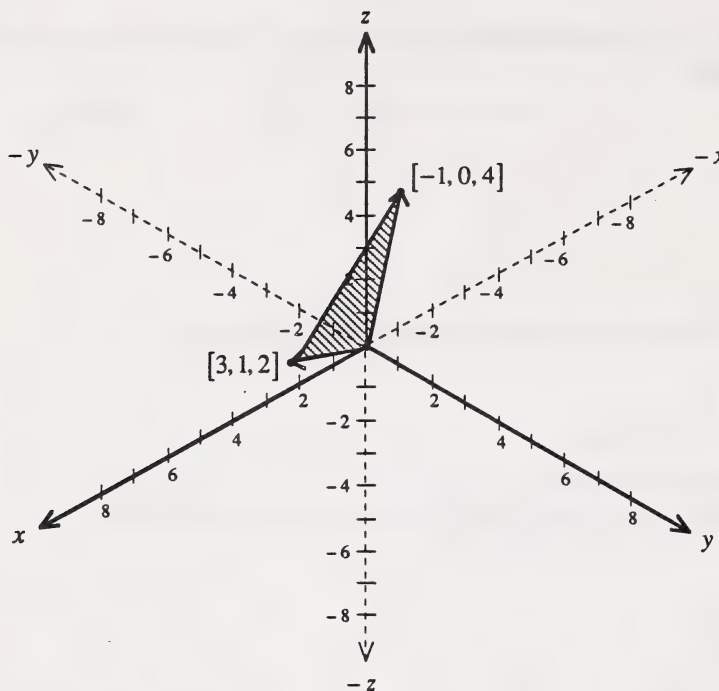
②

5. $\{k[-5, 0, -3] \mid k \in R\}$ determines the line in three-space through the origin. Find the coordinates of two points on the line.

Answers will vary.

4

6. The two position (basic) vectors $\vec{u} = [3, 1, 2]$ and $\vec{v} = [-1, 0, 4]$ determine a plane through the origin. Sketch the plane. (Shade the portion of the plane between the vectors.)



3

7. Are the vectors $[3, -5]$, $[9, -15]$, and $[-2, 7]$ collinear or coplanar?

$$[9, -15] = 3[3, -5]$$

These two vectors are collinear, but $[-2, 7]$ is not collinear with either of these.
Noncollinear vectors in V_2 are coplanar.

7

8. Are the vectors $[-1, -2, -3]$, $[0, 2, 1]$, and $[-2, 4, -2]$ collinear, coplanar, or noncoplanar?

No two vectors are collinear.

$$\begin{aligned}\text{Let } [-2, 4, -2] &= k[-1, -2, -3] + l[0, 2, 1] \\ &= [-k, -2k, -3k] + [0, 2l, l]\end{aligned}$$

$$-k = -2$$

$$k = 2$$

$$-2k + 2l = 4$$

$$-4 + 2l = 4$$

$$2l = 8$$

$$\therefore l = 4$$

Substitute $k = 2$ and $l = 4$ in $-3k + l = -2$ as a check.

$$LS = -3(2) + 4$$

$$= -2$$

$$\therefore LS = RS$$

Thus, $[-2, 4, -2]$ is a linear combination of the other two. They are coplanar.

6

9. Are the vectors $[3, 5, 7]$, $[0, -2, 1]$, and $[-1, 1, 3]$ collinear, coplanar, or noncoplanar?

By inspecting the vectors, no two vectors are collinear.

$$\begin{aligned}\text{Let } [3, 5, 7] &= k[0, -2, 1] + l[-1, 1, 3]. \\ &= [0, -2k, k] + [-l, l, 3l]\end{aligned}$$

$$3 = 0 - l \quad (1)$$

$$5 = -2k + l \quad (2)$$

$$7 = k + 3l \quad (3)$$

From (1), $l = -3$.

Substitute this in (2).

$$5 = -2k - 3$$

$$-2k = 5 + 3$$

$$k = -4$$

Substitute $l = -3$ and $k = -4$ in (3).

$$-4 + 3(-3) = -4 - 9$$

$$= -13$$

$$7 \neq -13$$

The vectors are noncoplanar.

Topic 4

_____ marks

N.L.C. - B.N.C.



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